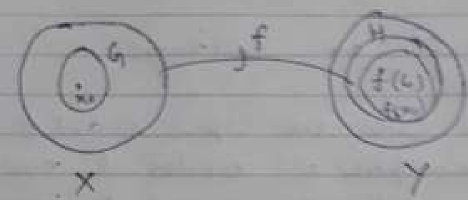
Video ⑤

1/5/2020 [illegible] Continuous functions

(i) Continuity :- Let $(X,\mathfrak{I})$ be a topological space and (Y,U) be another topological space.

Let $f:(X,\mathfrak{I}) \to (Y,U)$ be a map

This map f is said to be continuous at $x_0 \in X$ if given any U- open set H containing $f(x_0)$ $\exists$ a $\mathfrak{I}$- open set G containing x_0 s.t. $f(G) \subset H$



If this mapping is continuous at each $x_0 \in X$ then mapping is called a continuous mapping.

(ii) Sequentially continuous :- Let $f:(X,\mathfrak{I}) \to (Y,U)$ be a mapping.

f is sequentially continuous at $x_0 \in X$ if for every sequence

$$\langle x_n \in X : n \in N \rangle$$

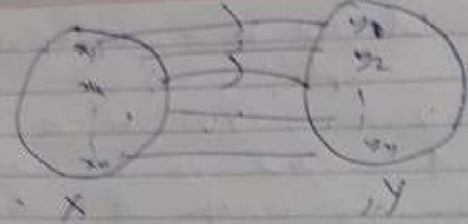
converges to x_0, the sequence $\langle f(x_n) \in Y : n \in N \rangle$ converges to $f(x_0)$, if $\Rightarrow$

$$x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$$

(iii) Homeomorphism or topological mapping:

A map ~~f~~ $f:(X,\mathfrak{I}) \to (Y,U)$ is said to be homeomorphism if

(i) f is one one onto



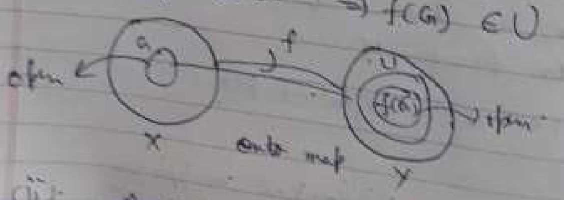
one - one onto

(ii) f and f^{-1} are continuous.

In this case the spaces X and Y are said to be homeomorphic or topological equivalent to one another and Y is said to be homeomorphic image of X.

(iv) Open and closed map :-

(i) open map :- A map $f:(X,\mathfrak{I})\to(Y,U)$ is called an open map if it maps open sets onto open sets i.e. if any $G\in\mathfrak{I}\Rightarrow f(G)\in U$



(ii) A map $f:(X,\mathfrak{I})\to(Y,U)$ is called a closed map if any $\mathfrak{I}$-closed set $F\Rightarrow f(F)$ is U-closed set